

On the renormalization of the one-pion exchange potential and the consistency of Weinberg's power counting

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Nogga, Timmermans and van Kolck recently argued that Weinberg's power counting in the few-nucleon sector is inconsistent and requires modifications. Their argument is based on the observed cutoff dependence of the nucleon-nucleon scattering amplitude calculated by solving the Lippmann-Schwinger equation with the regularized one-pion exchange potential and the cutoff Λ varied in the range $\Lambda = 2 \dots 20 \text{ fm}^{-1}$. In this paper we discuss the role the cutoff plays in the application of chiral effective field theory to the two-nucleon system and study carefully the cutoff-dependence of phase shifts and observables based on the one-pion exchange potential. We show that (i) there is no need to use the momentum-space cutoff larger than $\Lambda \sim 3 \text{ fm}^{-1}$; (ii) the neutron-proton low-energy data show no evidence for an inconsistency of Weinberg's power counting.

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I. INTRODUCTION

The use of the chiral effective Lagrangian in the analysis of the nuclear forces was pioneered by Weinberg [1, 2]. He realized that a perturbative treatment like in the pion or the pion-nucleon sector is not possible – the small nuclear binding energies require a non-perturbative resummation. Weinberg thus proposed to apply the power counting – the necessary ingredient to organize any effective field theory – to the effective nucleon-nucleon (NN) potential and generate the nuclear bound and scattering states utilizing this potential in a suitably regularized Lippmann-Schwinger equation. This approach has enjoyed a considerable success in the description of two-, three- and four-nucleon systems, for recent reviews see Ref. [3, 4]. However, soon after Weinberg's seminal work, some criticism has been raised about the consistency of this approach and proposals about modifications of the original power counting were made, most recently in Ref. [5] (that paper also contains a detailed discussion of earlier work on this subject and the corresponding references). In particular, it was argued that cutoff independence for a larger range of cutoff values than usually employed requires the promotion of certain next-to-leading order (NLO) operators to the leading order (LO). At this order, the effective potential is given by the one-pion exchange and two S-wave four-nucleon contact terms in Weinberg's power counting. In the modified counting of Ref. [5], three additional contact interactions in P- and D-waves are included in the LO potential. Here, we critically assess the statements made in that paper and draw some different conclusions, simply because we provide a *quantitative* measure of the theoretical accuracy following the seminal work of Lepage [6, 7]. In particular, the low-energy neutron-proton scattering data show no evidence for an inconsistency of Weinberg's original scheme. At this point, we should already stress that such investigations based on the LO potential are limited in their significance — the two-pion exchange potential that only enters at NLO is a very important part of the nuclear force. The well-known and important cancellations between pion- and ρ -exchange in the tensor channel could be interpreted as a signal that it also should be treated nonperturbatively. Even though effective field theory is the appropriate method to deal with the problem of nuclear forces, much has already been learned in less systematic schemes (like e.g. meson exchange models) and this knowledge needs to be accounted for.

Our manuscript is organized as follows. Section II contains a thorough discussion of the issues underlying regularization and renormalization of the Lippmann-Schwinger equation. In Sec. III, we analyze the cutoff dependence of the NN phase shifts, followed by a detailed comparison of observables calculated in the original Weinberg scheme and the

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modified power counting proposed in Ref. [5] in Sec. IV. A generalization of these issues to higher orders is discussed in Sec. V. We conclude and summarize our results in Sec. VI.

II. REGULARIZATION AND RENORMALIZATION OF THE LIPPMANN–SCHWINGER EQUATION

The leading-order two-nucleon force is given by the one-pion exchange potential (OPEP) accompanied by contact interactions (counterterms). The choice of counterterms is what makes the difference between the original [1, 2] and modified [5] Weinberg power counting schemes. The well-known expression for the LO one-pion exchange potential in momentum space reads

$$V_{1\pi}(\vec{q}) = -\left(\frac{g_A}{2F_\pi}\right)^2 \tau_1 \cdot \tau_2 \frac{\vec{\sigma}_1 \cdot \vec{q} \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^2 + M_\pi^2}, \quad (2.1)$$

where $\vec{q} = \vec{p}' - \vec{p}$ is the nucleon momentum transfer and $\vec{\sigma}_i$ (τ_i) are spin (isospin) matrices of the nucleon i . Further, g_A , F_π and M_π denote the axial-vector coupling constant, pion decay constant and pion mass, respectively. In this work, we use the values $F_\pi = 92.4$ MeV and $M_\pi = 138.03$ MeV. For the axial coupling constant g_A , we take the same value $g_A = 1.29$ as in our previous studies [8, 9] which accounts for the Goldberger–Treiman discrepancy and is, therefore, larger than the empirical value of $g_A = 1.267$. Both values for this LEC can be used at LO since the difference between them is a higher-order effect. In coordinate space, the OPEP is local and takes the form

$$V_{1\pi}(\vec{r}) = \left(\frac{g_A}{2F_\pi}\right)^2 \tau_1 \cdot \tau_2 \left[M_\pi^2 \frac{e^{-M_\pi r}}{12\pi r} \left(S_{12}(\hat{r}) \left(1 + \frac{3}{M_\pi r} + \frac{3}{(M_\pi r)^2} \right) + \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right) - \frac{1}{3} \vec{\sigma}_1 \cdot \vec{\sigma}_2 \delta^3(r) \right], \quad (2.2)$$

with \vec{r} being the relative distance between the nucleons, $r = |\vec{r}|$, $\hat{r} = \vec{r}/r$ and

$$S_{12} = 3 \vec{\sigma}_1 \cdot \hat{r} \vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2. \quad (2.3)$$

Notice that at higher orders in chiral EFT, the OPEP receives isospin-conserving and isospin-violating corrections, see e.g. [10], as well as the corrections of relativistic origin [11].

The nonrelativistic Lippmann–Schwinger (LS) equation for the half-off-energy shell T -matrix is given by

$$T(\vec{p}', \vec{p}) = V(\vec{p}', \vec{p}) + \int d^3 p'' V(\vec{p}', \vec{p}'') \frac{m}{\vec{p}^2 - \vec{p}''^2 + i\epsilon} T(\vec{p}'', \vec{p}), \quad (2.4)$$

with m being the nucleon mass, \vec{p} the on-shell point and V the potential. Clearly, the OPEP in Eq. (2.1) as well as contact interactions lead to ultraviolet divergencies in the LS equation which, therefore, needs to be regularized. This is usually achieved by multiplying the potential $V(\vec{p}, \vec{p}')$ with a regulator function f^Λ ,

$$V(\vec{p}, \vec{p}') \rightarrow f^\Lambda(\vec{p}) V(\vec{p}, \vec{p}') f^\Lambda(\vec{p}'). \quad (2.5)$$

An exponential regulator function

$$f^\Lambda(\vec{p}) = \exp[-\vec{p}^{2n}/\Lambda^{2n}] \quad (2.6)$$

with $n \geq 1$ is the commonly used choice. In particular, $f^\Lambda(\vec{p})$ with $n = 2$ was employed in Refs. [5, 12, 13] while $n = 3$ was used in [8, 9].

Before discussing the power counting, let us first take a closer look at the LS equation for the pure OPEP in the limit $\Lambda \rightarrow \infty$. It is easy to see that no ultraviolet divergences show up in the spin-singlet channels (with the exception of the 1S_0 partial wave), where the OPEP takes the form

$$V_{1\pi}^{S=0}(\vec{q}) = \left(\frac{g_A}{2F_\pi}\right)^2 \tau_1 \cdot \tau_2 \left(1 - \frac{M_\pi^2}{\vec{q}^2 + M_\pi^2} \right). \quad (2.7)$$

Despite the fact that the individual terms in the series resulting from iterating the LS equation (2.4) are ultraviolet divergent in all spin-triplet channels, the equation can still be solved nonperturbatively for those uncoupled partial

waves in which the singular part of the OPEP is repulsive. The resulting phase shifts are uniquely determined. The easiest way to see this is by looking at the radial Schrödinger equation which possesses a unique regular solution for repulsive singular potentials. On the contrary, it is ill-defined in the uncoupled channels where the $1/r^3$ -part of the OPEP is attractive. For these partial waves, both solutions of the radial Schrödinger equation go to zero at the origin while oscillating infinitely rapidly [14]. Imposing orthogonality of solutions allows to fix the discrete spectrum (which is unbound from below) and scattering states in terms of a single energy-independent phase factor [14], see also [15] for a comprehensive review on singular potentials. The case of coupled channels with $l, l' = j \pm 1$ can be reduced to the uncoupled one by decoupling the two Schrödinger equations in the short-distance region [16, 17]. For the OPEP, this yields the decoupled equations with an attractive and repulsive singular potentials for all j , so that one always needs to specify a single constant in order to uniquely determine both phase shifts and the mixing angle.

In practice, the LS equation (2.4) is usually studied for finite values of the cutoff Λ . Although the scattering amplitude based on the regularized OPEP is well-defined for any arbitrarily large but finite Λ , the limit $\Lambda \rightarrow \infty$ only exists in spin-singlet and repulsive uncoupled spin-triplet channels. In all coupled and uncoupled attractive spin-triplet channels the amplitude shows oscillatory behavior when Λ is taken to infinity. It is well known that the amplitude may be stabilized in each of these channels (for a finite energy) by introducing a single local counterterm¹ and adjusting its strength to reproduce e.g. the corresponding scattering length [5, 18, 19]. That is, a well-defined limit $\Lambda \rightarrow \infty$ of the scattering amplitude exists provided the strength $C(\Lambda)$ of the counterterm is fixed as described above. A similar observation was also made for the chiral potential at next-to-next-to-leading order in selected channels [20]. For a given partial wave, the above procedure provides the way to explicitly remove the scale associated with the ultraviolet cutoff and appears to be similar to the standard renormalization procedure applied in (perturbative) quantum field theory calculations. Despite this formal similarity, an important difference is given by the fact that the quantum mechanical approach described above is intrinsically nonperturbative. S-matrix elements depend nonanalytically on the strength of the OPEP for $\Lambda \rightarrow \infty$ in all spin-triplet channels [15] which negates a perturbative description in the weak-coupling limit [18].

We now turn to the actual subject of this study and consider the two-nucleon system at LO in chiral EFT based on Eq. (2.4) with the potential given by Eq. (2.1) and accompanied by local counterterms. Let us first ignore the potential interpretation difficulty due to the lack of the weak-coupling limit and require the scattering amplitude to be explicitly independent on the ultraviolet cutoff Λ . That is, we take the limit $\Lambda \rightarrow \infty$. As follows from the above discussion, an *infinite* number of local counterterms (more precisely, one term in each coupled and uncoupled attractive spin-triplet channels) are then needed in order to make the scattering amplitude well-defined. While such an approach still has predictive power for individual partial waves, the fact that an infinite number of parameters need to be determined to describe e.g. the cross section for a given energy makes it useless for practical applications. It has been argued in Ref. [5] that this complication might be avoided if the OPEP is treated in perturbation theory for high partial waves. We do not consider this argument as being relevant for the present discussion for the following reasons. First of all, it is disturbing that one needs to rely on a particular computational technique, namely the partial wave decomposition, in order to solve Eq. (2.4). This would rule out e.g. the three-dimensional approach of Ref. [21]. Secondly, if one decides to treat all partial waves for $l, l' \geq l_{\text{crit}}$ in perturbation theory, that is, if one assumes that l_{crit} is a large number, $l_{\text{crit}} \gg 1$, it is hard to argue why partial waves with $l, l' = l_{\text{crit}} - 1$ should not be treated perturbatively as well. Most important, however, is the fact the applicability of perturbation theory for high partial waves relies on the repulsive effect of the centrifugal barrier that goes like $l(l+1)/r^2$ and is only justified for regular potentials of a finite range. This certainly does not apply to attractive singular $1/r^3$ -potentials. The perturbative treatment of high partial waves is formally only justified if a finite cutoff is used in the LS equation. This possibility will be discussed below. In the limit of an infinitely large cutoff Λ , no predictions can be made for scattering observables (such as e.g. the cross section) if only a finite number of experimental data is available (i.e. if one does not know a priori that phase shifts are small for *all* high partial waves).

Clearly, the need to include infinitely many counterterms to absorb the ultraviolet divergences in the LS equation based on the OPEP is a direct consequence of the nonperturbative nature of the problem at hand. Contrary to standard chiral perturbation theory in the Goldstone Boson and single-nucleon sectors, where diagrams with a fixed number of loops contribute at a given order in the chiral expansion, infinite number of loops need to be taken into account at LO in the NN case. Taking the chiral limit of the OPEP, it is easy to verify based on the dimensional analysis that counterterms of the order Q^{2n} with $n = 1, 2, \dots$ and Q being the generic low-momentum scale are

¹ The term “local” means in this context that the corresponding nonregularized operator acts only near the origin.

required to absorb the divergences arising from $2n$ iterations of the LS equation, see [22] for a similar observation made based on the coordinate–space representation. While this difficulty might first appear as an indication of the uselessness of the chiral EFT approach in the few–nucleon sector, it is actually not. Contrary to renormalizable field theories like QCD, EFTs (such as e.g. pionless or chiral EFT) are low–energy expansions with the finite radius of convergence representing their intrinsic ultraviolet cutoff $\bar{\Lambda}$. As a consequence, no improvement in the description of the data can, generally, be expected increasing Λ beyond the pertinent hard scale $\bar{\Lambda}$. Keeping Λ finite does, therefore, not mean any loss of generality (provided the error due to Λ being finite is within the theoretical uncertainty at a given order) neither does it violate the requirement on the EFT of being a systematically improvable low–energy approximation to a given underlying theory.

The above arguments make it clear that nonrenormalizability of the OPEP in Eq. (2.1) is not a fundamental problem but rather an artifact of a particular choice for the large–momentum (or, equivalently, short–distance) behavior of the OPEP out of infinitely many *equivalent* (from the viewpoint of chiral EFT) ones. For example, including the corrections to account for the proper normalization of the nonrelativistic single–nucleon states yields the expression for the OPEP of the form

$$\frac{m}{E} V_{1\pi}(\vec{q}) \frac{m}{E} = \frac{m}{\sqrt{\vec{p}'^2 + m^2}} V_{1\pi}(\vec{q}) \frac{m}{\sqrt{\vec{p}^2 + m^2}}, \quad (2.8)$$

which does not lead to ultraviolet divergences in the LS equation. Despite the different properties in the ultraviolet region, both forms in Eq. (2.1) and Eq. (2.8) are, of course, equally valid (as well as the expression resulting from the p/m –expansion of Eq. (2.8)) as the differences between them are of higher orders in the EFT expansion. The behavior of the potential for momenta larger than $\bar{\Lambda}$ is not constrained in chiral EFT and can always be modified (via inclusion of a set of higher–order terms) to make the LS equation free from ultraviolet divergences, see e.g. Eq. 2.5.

Having accepted the viewpoint that taking the limit $\Lambda \rightarrow \infty$ does not represent a fundamental (and even useful) requirement on chiral EFT in the few–nucleon sector, the following two issues are important for practical applications and will be dealt with in the next sections:

- how to choose the value of Λ to make the EFT expansion most efficient ?
- what counterterms need to be taken into account, i.e. what are the implications for the power counting ?

III. CUTOFF DEPENDENCE OF THE NN PHASE SHIFTS

In the past decade, the NN system has been explored within the chiral EFT framework in Weinberg’s formulation [1, 2] based on the finite momentum–space cutoff Λ in the range 300...700 MeV in [23], 450...650 (600) MeV in [8] ([9]), 500 MeV in [24], 500...1000 MeV in [12, 25]. In addition, much larger cutoff values upto ~ 4 GeV were used in the LO calculation of Ref. [5], where certain contact terms of higher–orders in the original Weinberg power counting scheme have been included. Finally, the case of an infinite cutoff has been studied in Refs. [17, 22, 26, 27] using the boundary condition regularization prescription. This large variation in values for Λ adopted by different authors raises the important question on the criteria for choosing the cutoff. Clearly, taking Λ too small, i.e. much smaller than the separation scale $\bar{\Lambda}$, will remove the truly long–distance physics and reduce the predictive power of the theory. On the other hand, it is argued in [6, 7] that too large values for Λ result in a highly nonlinear behavior and should be avoided as well, see also [28] and [29]. In order to clarify this last statement, let us make a rather plausible assumption that the low–energy two–nucleon dynamics can be described within the framework of quantum mechanics based on the Hamilton operator given by the kinetic energy and a non–singular NN potential (in practice, one can think about e.g. the Nijm I, II or Reid 93 potentials [30]). Keeping the point–like static OPEP in Eq. (2.1) as the leading approximation of the long–range part of the underlying interaction, it is certainly preferable to adopt the smallest (yet acceptable) value for the cutoff, that is, $\Lambda \sim \bar{\Lambda}$. Increasing Λ substantially beyond this scale would result in incorporating the incorrect singular short–range part of the OPEP and, as a consequence, would require the inclusion of a large number of counterterms to compensate for it.² In the extreme limit $\Lambda \rightarrow \infty$, an infinite number of counterterms are necessary.

² In some loose sense, this might be viewed as a violation of the “naturalness assumption”.

Based on the above arguments, one might expect the ultraviolet cutoff of the order of the chiral symmetry breaking scale, i.e. $\Lambda_\chi \sim 1$ GeV, to be a reasonable choice. On the other hand, the radius of convergence of the chiral expansion for the NN scattering amplitude cannot exceed $|\vec{p}| \sim M_\rho/2 \sim 400$ MeV since e.g. the left-hand cut associated with the ρ -meson exchange cannot be properly reproduced in chiral EFT at any finite order. A (numerically) similar estimation follows also from the fact that chiral EFT without explicit pions is only applicable below the pion production threshold. These arguments suggest that even significantly smaller values of Λ in Eq. (2.6) might be sufficient. A detailed discussion on the choice of ultraviolet cutoff and its role in renormalization of the Schrödinger equation is given by Lepage [6, 7]. He argued that the coordinate-space (momentum-space) cutoff should not be decreased (increased) beyond the critical value, after which the description of the data stops to improve. Taking the cutoff near this critical point is the most efficient choice.

In the present work, we follow a similar path and determine *the minimal acceptable value for the cutoff Λ as the one for which the error due to keeping Λ finite is within the theoretical uncertainty of the LO approximation*. To that aim, we use the modified power counting scheme introduced by Nogga et al. [5] which is compatible with the variation of Λ in a large range we are interested in. We repeat the analysis of Ref. [5] and consider the same partial waves based on the OPEP in Eq. (2.1) accompanied by the counterterms and multiplied with the regulator function $f^\Lambda(\vec{p})$ in Eq. (2.6). We use the cutoff $\Lambda < 20 \text{ fm}^{-1} \sim 4$ GeV and include the counterterms following the rules of Ref. [5], see also the discussion in section II. In particular, one contact interaction is taken into account in the 1S_0 partial wave while no counterterms are included in the spin-singlet channels 1P_1 , 1D_2 , 1F_3 and 1G_4 as well as in the uncoupled spin-triplet partial waves 3P_1 and 3F_3 where the tensor part of the OPEP is repulsive. Further, one contact interaction is included in each of the coupled channels 3S_1 - 3D_1 and 3P_2 - 3F_2 ³ as well as in the uncoupled spin-triplet channels 3P_0 and 3D_2 , where the tensor part of the OPEP is attractive. Specifically, the following contact interactions are incorporated:

$$\begin{aligned} \langle S | V_{\text{cont}} | S \rangle &= C_S, \quad S = \{^1S_0, ^3S_1\}, \\ \langle P | V_{\text{cont}} | P \rangle &= C_P p p', \quad P = \{^3P_0, ^3P_2\}, \\ \langle D | V_{\text{cont}} | D \rangle &= C_D p^2 p'^2, \quad D = ^3D_2. \end{aligned} \quad (3.9)$$

For each value of the cutoff Λ in Eq. 2.6, the LECs C_i are determined by fitting the corresponding phase shifts to the Nijmegen PWA [31] in the low-energy region $E_{\text{lab}} < 10$ MeV. Our way of fixing the LECs is, therefore, somewhat different to the one adopted in [5] and is more in spirit of Ref. [22, 26, 27]. The results for the phase shifts and mixing angles are presented in Fig. 1 for two choices of the cutoff: $\Lambda = 3$ and 20 fm^{-1} . We also show the results for the case of the 3D_3 - 3G_3 coupled channels, where we do not include counterterms following Ref. [5]. The small differences between our results and the ones of Nogga et al. may be attributed to the different fitting procedure, the choice of the regulator function and the adopted value for g_A . Notice that we also obtain similar values for the deeply bound states in the attractive spin-triplet channels. In particular, for $\Lambda = 20 \text{ fm}^{-1}$ we find $E_b = 1.69$ and 34.27 GeV in the 3S_1 - 3D_1 , 0.22 and 21.48 GeV in the 3P_0 and 0.26 GeV in the 3D_2 . While no deeply bound states appear in the 3P_2 - 3F_2 and 3D_3 - 3G_3 for $\Lambda \leq 20 \text{ fm}^{-1}$, one should keep in mind that they necessarily show up for larger cutoff values. We further emphasize that the binding energies of the first spurious states in the 3P_0 and 3D_2 channels are consistent with the above estimation for the separation scale $\bar{\Lambda}$.

We are now in the position to determine the minimal acceptable value for Λ . To that aim, we vary the cutoff in the range from $\Lambda = 1 \text{ fm}^{-1}$ up to $\Lambda = \infty$ and study carefully the cutoff dependence of the phase shifts. Notice that since unstable channels are stabilized by adding the appropriate counterterms and tuning their strengths as described above, all phase shifts and mixing angles are expected to reach stable values as Λ goes to infinity. Clearly, as Λ is taken very small, $\Lambda \sim 1 \text{ fm}^{-1}$, one expects large errors in the description of the data. Increasing the cutoff values the results are expected to improve until Λ reaches the critical point where missing higher-order contributions to the potential due to e.g. 2π -exchange become the dominant source of errors. Starting from this point, the error due to keeping Λ finite is within the theoretical uncertainty of the LO approximation. These qualitative expectations are confirmed by explicit calculations as shown in Fig. 2, where errors in phase shifts in the uncoupled channels are depicted. The results for $\Lambda > 20 \text{ fm}^{-1}$ become nearly cutoff-independent and are not shown for this reason. Clearly the theoretical uncertainty at a given order in the EFT expansion cannot be smaller than the deviation from the data. The lower bound for the LO theoretical uncertainty in a partial wave i is, therefore, given by $\lim_{\Lambda \rightarrow \infty} |\Delta\delta^i|$ and can be read off from Fig. 2. One observes that already for $\Lambda \gtrsim 3 \text{ fm}^{-1}$ the error due to keeping Λ finite is within the theoretical

³ In a coupled channel with the total angular momentum j , we take into account the leading counterterm in the partial wave with $l = l' = j - 1$.

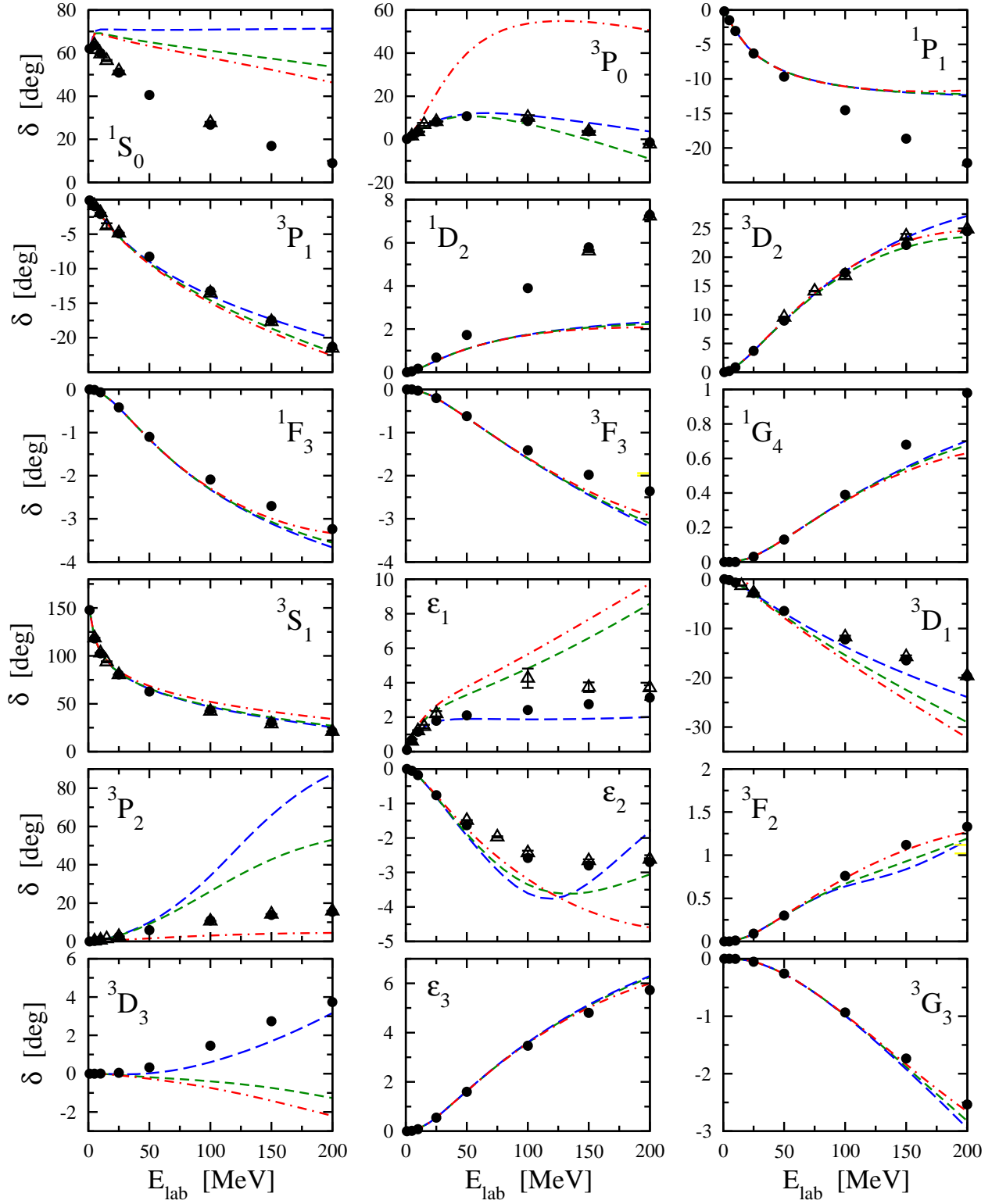


FIG. 1: Comparison of the np phase shifts at LO in the original and modified Weinberg power counting scheme to the Nijmegen [31, 32] (solid dots) and Virginia Tech single-energy PWA [33] (open triangles). Dot-dashed lines correspond to the original Weinberg power counting with $\Lambda = 2.5 \text{ fm}^{-1}$ while short-dashed (long-dashed) lines depict the results obtained within the modified counting scheme with $\Lambda = 3 \text{ fm}^{-1}$ ($\Lambda = 20 \text{ fm}^{-1}$).

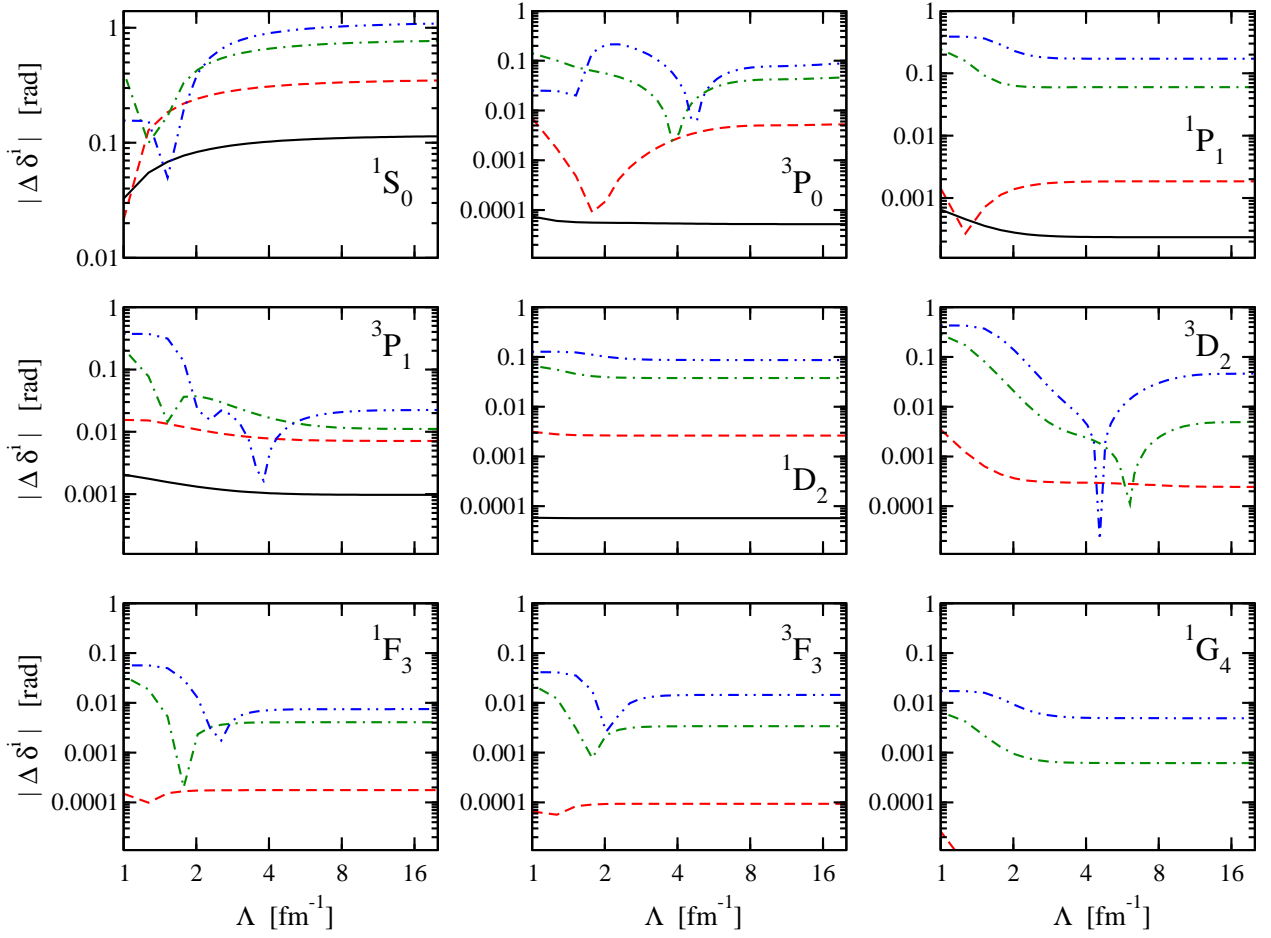


FIG. 2: Error in phase shifts in the uncoupled channels versus the cutoff Λ in the Lippmann-Schwinger equation. Results are given for laboratory energies of 5 MeV (solid line), 25 MeV (short-dashed line), 100 MeV (dot-dashed line) and 200 MeV (dot-dot-dashed line).

uncertainty of the LO approximation. The only exception is given by the 3P_1 phase shift, where the critical value of the cutoff $\Lambda \sim 4 - 5 \text{ fm}^{-1}$ is somewhat larger. On the other hand, even a smaller cutoff value $\Lambda \sim 2 \text{ fm}^{-1}$ seems to be sufficient in the 1S_0 , 1F_3 and 3F_3 partial wave.

We now turn to the coupled channels. Here we have decided to look at the error in the T-matrix elements rather than in the corresponding phase shifts and mixing angles in order to avoid the ambiguity associated with different parametrizations of the coupled-channel S-matrix elements. More precisely, we consider dimensionless on-the-energy-shell T-matrix elements $T_{ll'}^{sj}$ related to S-matrix via

$$T_{ll'}^{sj} = S_{ll'}^{sj} - \delta_{ll'} . \quad (3.10)$$

Our results for the 3S_1 - 3D_1 and 3P_2 - 3F_2 channels are shown in Fig. 3. The critical values of the cutoff are $\Lambda \sim 2 \text{ fm}^{-1}$ for T_{11}^{12} , T_{13}^{12} and T_{33}^{12} , $\Lambda \sim 3 \text{ fm}^{-1}$ for T_{11}^{00} and $\Lambda \sim 4 - 5 \text{ fm}^{-1}$ for T_{11}^{02} and T_{11}^{22} . We have also looked at the deuteron binding energy, see Fig. 4, which shows a similar pattern with the critical cutoff value $\Lambda \sim 3 \text{ fm}^{-1}$.

The case of the 3D_3 - 3G_3 channel requires a special consideration. Due to the relatively mild cutoff dependence for $\Lambda < 20 \text{ fm}^{-1}$, no counterterm has been included in this channel in the analysis of Ref. [5]. In Fig. 5 we show the dependence of the 3D_3 phase shift and of the binding energies of the spurious bound states on the cutoff in the large range for the values of the cutoff Λ . The cutoff dependence of the phase shift is indeed rather small for $\Lambda < 20 \text{ fm}^{-1}$ at low energies but becomes significant at $E_{\text{lab}} = 200 \text{ MeV}$. The first spurious bound state is generated in this channel at $\Lambda \sim 29 \text{ fm}^{-1}$. In order to study the behavior of the phase shifts in the limit $\Lambda \rightarrow \infty$ in these partial waves, we proceed in the same way as we did for the two other coupled channels and introduce a counterterm as shown in

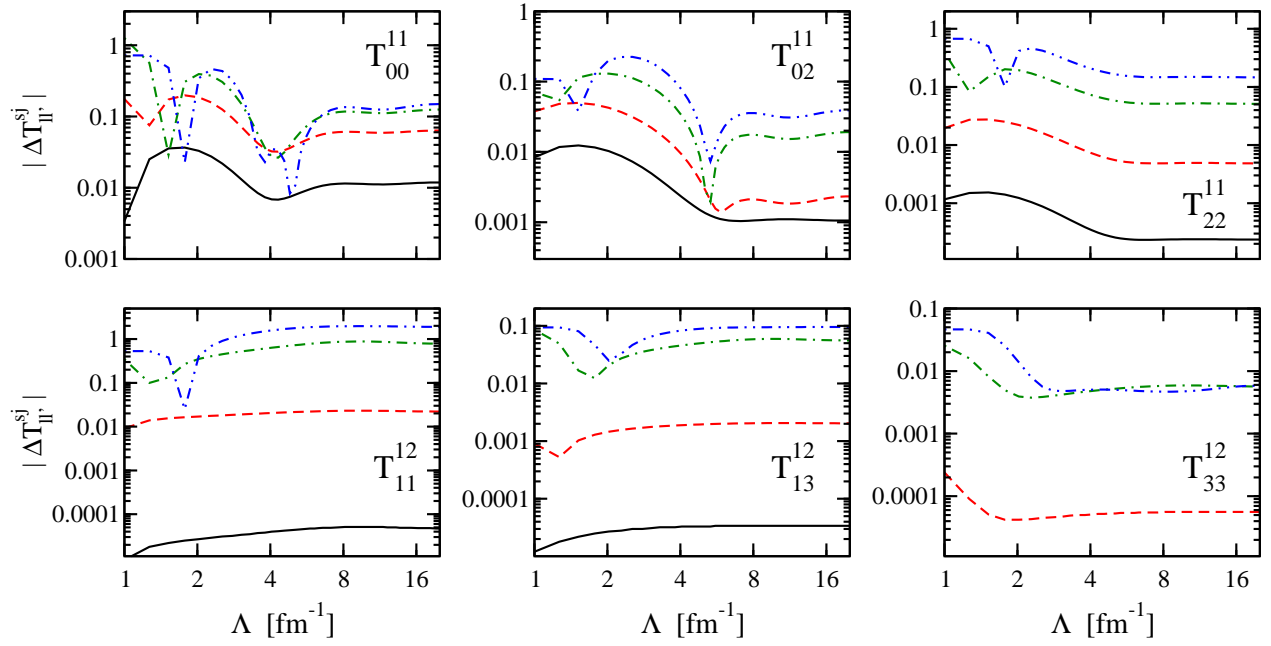


FIG. 3: Error in T-matrix elements in the coupled channels versus the cutoff Λ in the Lippmann–Schwinger equation. Results are given for laboratory energies of 5 MeV (solid line), 25 MeV (short-dashed line), 100 MeV (dot-dashed line) and 200 MeV (dot-dot-dashed line).

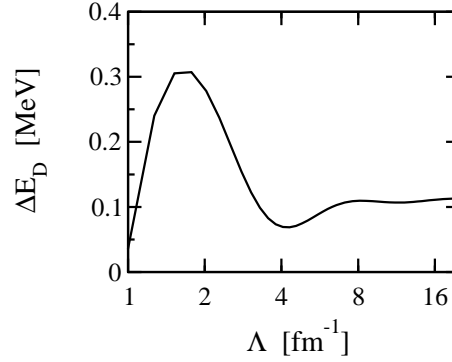


FIG. 4: Error in the deuteron binding energy as a function of the cutoff Λ .

the last line of Eq. 3.9. The corresponding LEC has to be determined for a given value of Λ from a fit to the 3D_3 phase shift in the low-energy region. We found that the 3D_3 phase shift starts to increase rapidly at energies of the order $E_{\text{lab}} \sim 50$ leading to dramatic deviations from the data if the fit is performed in our standard range $E_{\text{lab}} < 10$ MeV. This behavior can probably be attributed to an unnaturally small value of this phase shift close to threshold. In order to obtain meaningful results, we have, therefore, decided to perform a global fit in the large energy region up to $E_{\text{lab}} = 200$ MeV. Our results for Λ -dependence of the T-matrix elements obtained in such a way are depicted in Fig. 6. Similarly to the previously discussed channels, keeping Λ finite does not represent the main source of uncertainty at LO already for $\Lambda \sim 2 \text{ fm}^{-1}$. The only exception is given by the 3D_3 phase shift at the highest energy considered, $E_{\text{lab}} = 200$ MeV, where the results at this order cannot be trusted anyway.

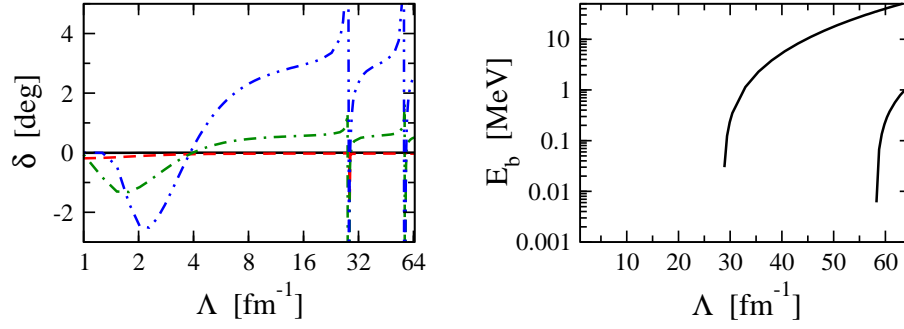


FIG. 5: Cutoff dependence of the 3D_3 phase shift (left panel) and the binding energies of the spurious bound states (right panel). Results for the phase shift are given for laboratory energies of 5 MeV (solid line), 25 MeV (short-dashed line), 100 MeV (dot-dashed line) and 200 MeV (dot-dot-dashed line).

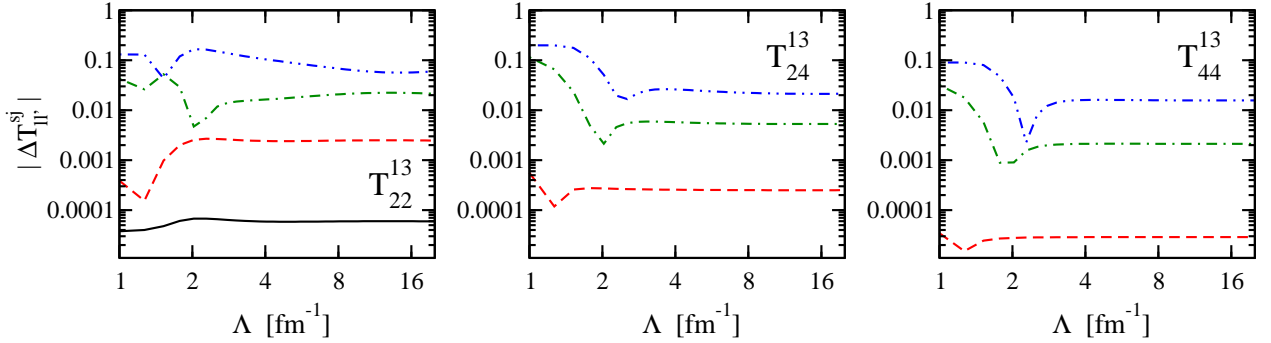


FIG. 6: Error in T-matrix elements in the 3D_3 – 3G_3 channel versus the cutoff Λ in the Lippmann–Schwinger equation. Results are given for laboratory energies of 5 MeV (solid line), 25 MeV (short-dashed line), 100 MeV (dot-dashed line) and 200 MeV (dot-dot-dashed line).

IV. NN OBSERVABLES: WEINBERG VERSUS NOGGA–TIMMERMANS–VAN KOLCK

Having convinced ourselves that a finite cutoff of the order $\Lambda \sim 2 - 3 \text{ fm}^{-1}$ is sufficient in most partial waves to ensure that the cutoff dependence is within the actual theoretical uncertainty of the LO approximation, we are now in the position to study the implications for the power counting. To be specific, we consider a selection of np scattering observables at LO in chiral EFT with the finite cutoff chosen as described above within the two different power counting schemes: the one of Weinberg (W) and the one of Nogga et al. (NTvK). The potential in the first case is given by the OPEP accompanied by two contact terms without derivatives that contribute to the 1S_0 and 3S_1 partial waves. For the cutoff we use the value $\Lambda = 2.5 \text{ fm}^{-1}$ which is close to the central value used in Refs.[8, 9] and has also been adopted in [24]. For the calculation based on the NTvK counting, we choose $\Lambda = 3 \text{ fm}^{-1}$ and include, in addition to the two S-wave contact interactions also counterterms in the 3P_0 , 3P_2 and 3D_2 channels as shown in Eq. (3.9). For the sake of completeness, we will also show the results in the NtvK counting scheme corresponding to $\Lambda = 20 \text{ fm}^{-1}$, the largest cutoff considered in [5].

The results for the differential cross section, vector analyzing power A_y , depolarization observables D and A and spin correlation parameters A_{xx} and A_{yy} for energies $E_{\text{lab}} = 5, 25, 100$ and 200 MeV are shown in Figs.7–10. We use the same convention for observables as the one adopted by the Nijmegen group [32]. For a precise definition of various nucleon–nucleon scattering observables see e.g. the appendix of Ref. [34]. The results obtained at LO in chiral EFT have to be compared to the ones obtained based on np phase shifts from Nijmegen PWA [32] which should be regarded as experimental data and are shown by the solid lines. In the present study, we neglect the electromagnetic interaction between the neutron and proton. To visualize the effects due to the (neglected) magnetic moment interaction, we also show by the dotted line the results for various observables calculated by the Nijmegen group [32] which include the magnetic moment interaction. The only significant effects are observed for A_y at small scattering angles and energies

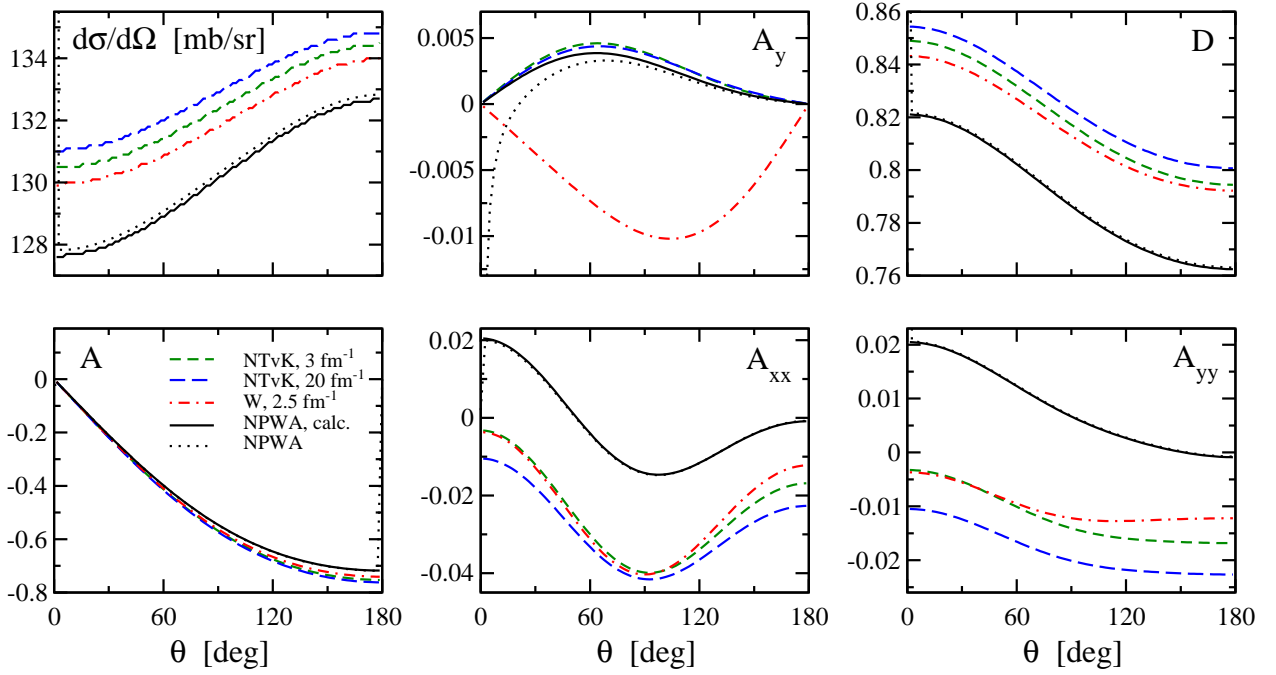


FIG. 7: np differential cross section $d\sigma/d\Omega$, vector analyzing power A_y , polarization transfer coefficients D and A and spin correlation parameters A_{xx} and A_{yy} versus the scattering angle θ for laboratory energy of 5 MeV. Short-dashed (long-dashed) lines show the results obtained in the modified Weinberg power counting scheme with the cutoff $\Lambda = 3 \text{ fm}^{-1}$ ($\Lambda = 20 \text{ fm}^{-1}$) while dot-dashed line depicts the results based on the original Weinberg power counting. Solid lines shows the observables calculated based on phase shifts from Nijmegen PWA while dotted line is the true Nijmegen PWA result from [32].

as well as for differential cross section in the forward direction. At the smallest energy, $E_{\text{lab}} = 5 \text{ MeV}$, all three EFT calculations lead to similar results for $d\sigma/d\Omega$, D and A . In the case of vector analyzing power A_y , the results based on the NTvK counting are consistent with the data while the ones obtained in the W scheme do not even reproduce the correct sign. This failure of the W counting can be traced back to the relatively large error in the 3P_0 partial wave at low energy, see Fig.11, and to the well-known extreme sensitivity of A_y to the triplet P-waves. In the NTvK counting, the 3P_0 is, by construction, much better reproduced at low energy due to the presence of the additional counter term. We emphasize, however, that despite the large relative error, the absolute error for A_y in the W scheme is still small as a consequence of its tiny size. Similarly, large relative deviations of the order of 100% are observed in both W and NTvK counting schemes for A_{xx} and A_{yy} , which are also rather small at this energy.

The results at $E_{\text{lab}} = 25 \text{ MeV}$ are shown in Fig. 8. The differential cross section is better reproduced in the NTvK scheme in the forward and in the W scheme in the backward angles. The situation with the vector analyzing power, which is still very small numerically, is similar to the one at $E_{\text{lab}} = 5 \text{ MeV}$. On the other hand the spin correlation parameters A_{xx} and A_{yy} , which have a more natural size, are now much better reproduced. Notice that for D , A , A_{xx} and A_{yy} , the results obtained in the W counting show a significantly better agreement with the data compared to the ones based on the NTvK scheme. We also emphasize that increasing the cutoff in the NTvK scheme from 3 to 20 fm^{-1} does not lead to improvement, which is consistent with our findings in section III.

At $E_{\text{lab}} = 100 \text{ MeV}$, one observes moderate (large) deviations for the differential cross section in the W (NTvK) counting, see Fig. 9. The description of the data in the NTvK scheme gets worse with increasing Λ . The situation with the analyzing power is now opposite to the one observed at the two smaller energies. It is best described in the W scheme, while the prediction in the NtvK scheme deviates strongly from the data, especially in the case of the large cutoff. The results for D , A_{xx} and A_{yy} are comparable in both approaches while the description of A is slightly better in the W counting.

Finally, at $E_{\text{lab}} = 200 \text{ MeV}$, the largest energy considered, the results start to diverge even for the differential cross section, see Fig 10. Clearly, no quantitative agreement with the data can be expected at this high energy at LO in the chiral expansion. The prediction for $d\sigma/d\Omega$ based on the W counting turns out to be in a much better agreement

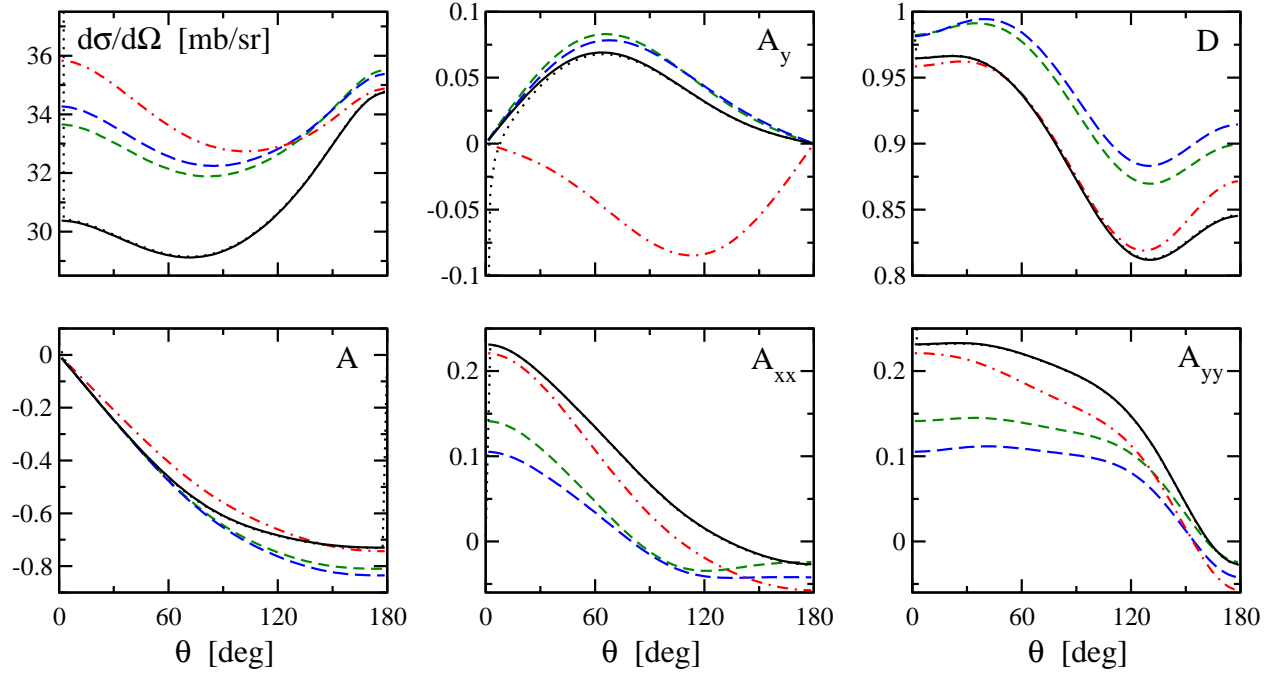


FIG. 8: np differential cross section $d\sigma/d\Omega$, vector analyzing power A_y , polarization transfer coefficients D and A and spin correlation parameters A_{xx} and A_{yy} versus the scattering angle θ for laboratory energy of 25 MeV. For remaining notation see Fig. 7.

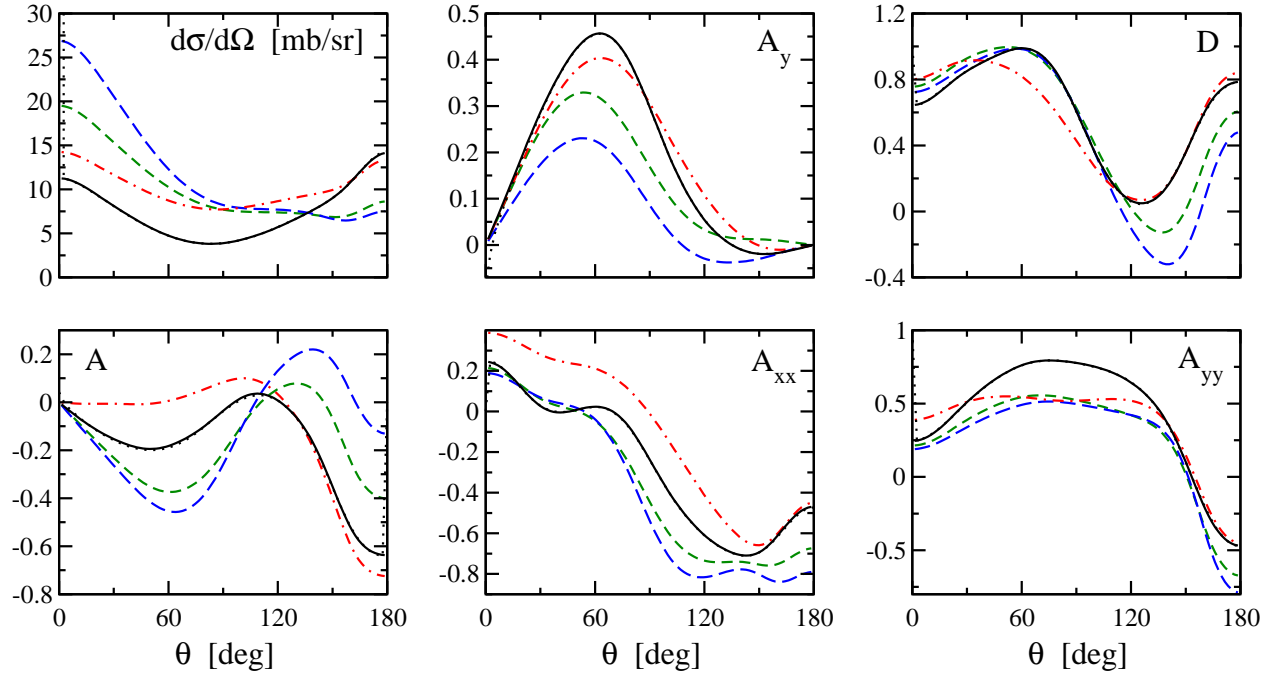


FIG. 9: np differential cross section $d\sigma/d\Omega$, vector analyzing power A_y , polarization transfer coefficients D and A and spin correlation parameters A_{xx} and A_{yy} versus the scattering angle θ for laboratory energy of 100 MeV. For remaining notation see Fig. 7.

with the data compared to the NTvK scheme. The results for all other observables are of a similar quality and show mostly a qualitative agreement with the data.

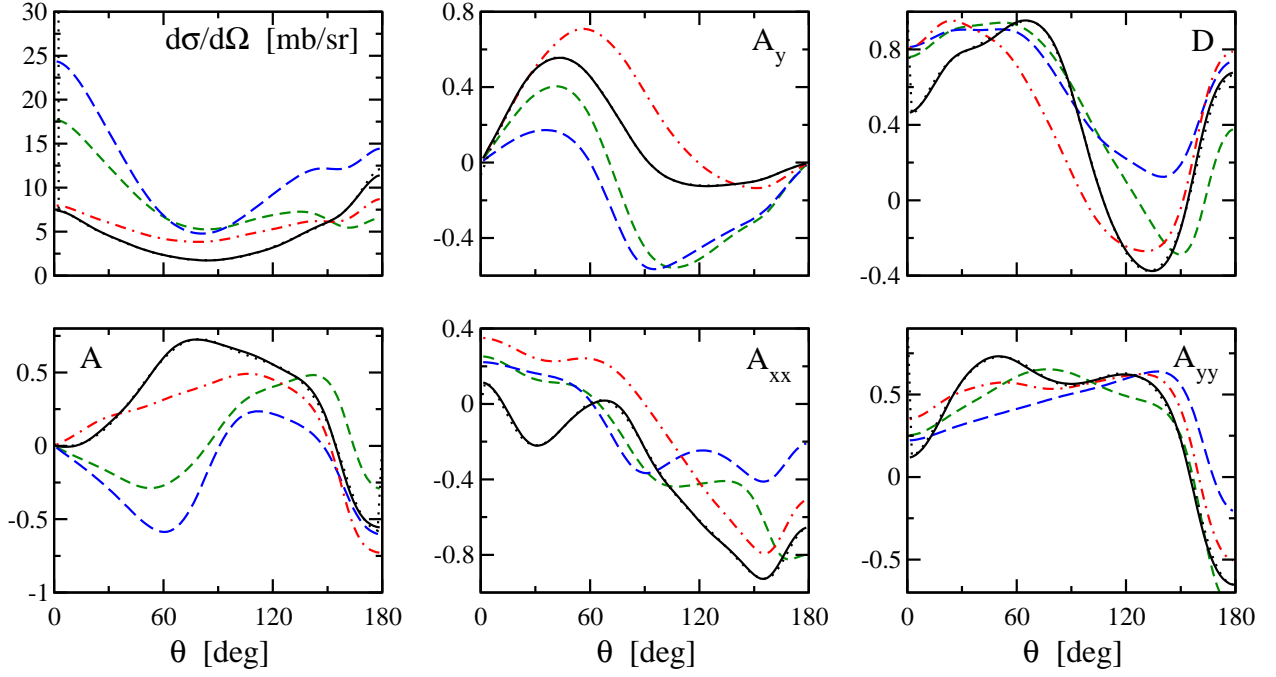


FIG. 10: np differential cross section $d\sigma/d\Omega$, vector analyzing power A_y , polarization transfer coefficients D and A and spin correlation parameters A_{xx} and A_{yy} versus the scattering angle θ for laboratory energy of 200 MeV. For remaining notation see Fig. 7.

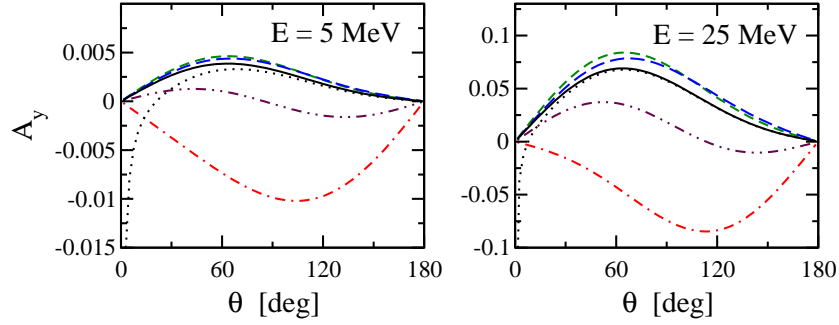


FIG. 11: np vector analyzing power A_y at $E_{\text{lab}} = 5$ and 25 MeV. The double-dotted-dashed line shows the result in the W scheme with the 3P_0 partial taken from the Nijmegen PWA [32]. For remaining notation see Fig. 7.

Based on the above results, we conclude that both W and NTVK counting schemes lead to a similar description of the np scattering data for the appropriately chosen cutoff (i.e. $\Lambda \sim 3 \text{ fm}^{-1}$). The modification of the original W counting by promoting certain higher-order contact interactions to LO in the NTVK scheme seems not to be supported by the data. In addition, we confirm our findings in the previous section that increasing Λ in the calculations based on the NTVK scheme does not yield any improvement.

V. GENERALIZATION TO HIGHER ORDERS

Let us now briefly discuss some possible ways to include higher-order corrections in the NTVK scheme, see Ref. [27] for a much more detailed discussion. A straightforward generalization by solving the LS equation with the potential which includes higher-order chiral corrections is, in general, not possible. To see that consider e.g. the 3D_2 partial wave at NLO. Since the OPEP is singular and attractive in this channel, the LO potential in the NTVK includes, in

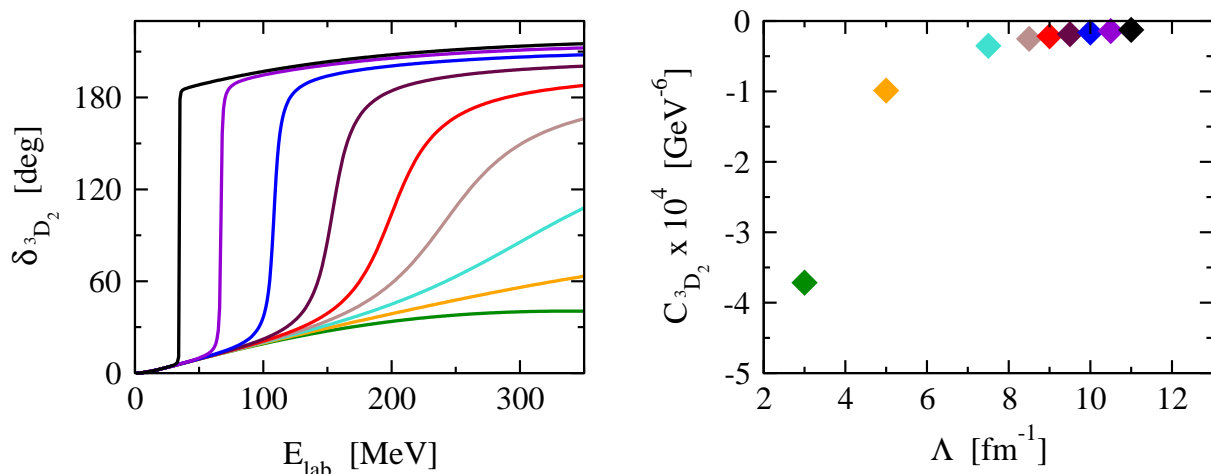


FIG. 12: 3D_2 phase shift obtained by solving the LS equation with the one-, the leading two-pion exchange potential and the counterterm from Eq. (3.9) (left panel). The lines (from bottom to top) correspond to $\Lambda = 3.0, 5.5, 7.5, 8.5, 9.0, 9.5, 10.0, 10.5$ and 11.0 fm^{-1} . In the right the cutoff dependence of the corresponding LEC versus the cutoff Λ is shown.

addition, one counterterm as shown in Eq. (3.9). At NLO, the long-range part of the potential receives the correction due to the leading 2π -exchange, which is repulsive in this channel and behaves at short distances as $1/r^5$. We now follow the approach of section III and determine the value of $C_{^3D_2}$ as a function of Λ from a fit to the Nijmegen phase shift for $E_{\text{lab}} < 10 \text{ MeV}$. As shown in Fig.12, the nonperturbative inclusion of the leading 2π -exchange potential leads to the appearance of a resonance, which gets sharper and moves to lower energies with increasing the cutoff Λ . This is consistent with the observation made by Pavon Valderrama and Ruiz Arriola [22, 27] in the case $\Lambda \rightarrow \infty$.⁴

The above mentioned difficulties clearly demonstrate that a generalization of the NTVK scheme to include higher-order chiral corrections is not straightforward. Possible scenarios include:

- i) One may try to promote additional higher-order counterterms in order to remove resonances from the low-energy region. This scenario seems to be in conflict with the $\Lambda \rightarrow \infty$ formulation based on the boundary condition approach of [22, 27].
- ii) The approach by Pavon Valderrama and Ruiz Arriola [22, 27] allows to avoid the above-mentioned problems by demoting the counter terms in the singular repulsive channels. Such an approach, however, appears to be not systematically improvable in the sense that it does not provide a systematic way to include short-range physics beyond the pion-exchange contributions.
- iii) Finally, there is a more natural solution based upon a perturbative treatment of the higher-order corrections, which reduces to the distorted wave perturbation theory using the exact LO wave functions [5, 35]. As pointed out in [27], this would require a large number of counterterms, which raises the concern about its predictive power. It remains to be seen whether such a scheme will lead to a perturbative expansion for the nucleon-nucleon observables.

VI. SUMMARY AND CONCLUSIONS

We summarize by repeating the logic we have followed in this manuscript:

⁴ Notice that while the above mentioned difficulties are absent at N²LO due to the strong attraction caused by the isoscalar central part of the subleading TPEP [22, 27], the appearance of the singular repulsive potentials is unavoidable at higher orders in the chiral expansion.

- i) The LS (or Schrödinger) equation with the point-like static OPEP is ill-defined (ultraviolet divergent). An infinite number of counterterms are needed in the limit $\Lambda \rightarrow \infty$. Treating the high partial waves in perturbation theory cannot be justified if one does not make assumptions about the range of applicability of the point-like OPEP, i.e. if one does not implicitly introduce a finite UV cutoff. The approach with $\Lambda \rightarrow \infty$ seems therefore to be useless if one wants to consider observables rather than specific partial waves.
- ii) Of course, this is not a conceptual problem in an EFT. The potential description is only valid for small momenta/energies. There are infinitely many forms of the OPEP potential which differ from each other by short-range terms and that are equivalent from the viewpoint of EFT. Choosing an appropriate short-range extension corresponds to using a finite cutoff. A striking example is the relativistically corrected expression for OPEP, cf. Eq. (2.8) – it does not lead to divergences in the LS equation.
- iii) If one agrees to work with a finite Λ , then the question is how to choose it. Our answer is: *Any value of Λ is acceptable if the error associated to its finite value is within the theoretical uncertainty at a given order.* This is a well-defined criterion in contrast to the one used in Ref. [5] (called “strong” or “mild” cutoff dependence). We have studied the NN phase shifts based on the OPEP and found that starting from $\Lambda \sim 3 \text{ fm}^{-1}$, the error due to keeping Λ finite is within the theoretical uncertainty at LO.
- iv) It is advantageous for many reasons to keep Λ small (yet acceptable), see [6, 7, 29, 36].
- v) Having convinced ourselves that $\Lambda \sim 3 \text{ fm}^{-1}$ is acceptable, the question is: what counterterms need to be included? The choices are: i) W-counting (based on naive dimensional analysis); ii) NTVK counting (based on the asymptotics in the limit $\Lambda \rightarrow \infty$). We have compared the predictions for various observables in these two counting schemes and found that the promotion of certain counterterms to LO as suggested in the NTVK scheme is not supported by the data (except for the case of A_y as discussed above).

Finally, we list a few advantages of the W-approach with a suitably chosen finite cutoff Λ :

- i) The potentials constructed so far are fairly smooth for the ranges of cutoffs considered, this implies that few- and many-body calculations are simpler to perform since one does not have to deal explicitly with spurious deeply bound states. In particular, this leads to “more perturbative” many-body calculations, see e.g. the discussion in Ref. [36].
- ii) At a given order, one has less parameters, that means that the predictive power is higher. Furthermore, we stress again that the choice of the cutoff assumes the knowledge of the separation scale.
- iii) The analysis of the nuclear forces as a function of the quark masses can be done straightforwardly and is much more transparent than in any modified power counting scheme.
- iv) The Weinberg approach can easily be extended to systems for N ($N > 2$) nucleons based on the N -nucleon Schrödinger equation. These forces are also amenable to a lattice formulation, as recently shown in Ref. [37].
- v) Finally, one might pose the question whether the mild explicit cutoff dependence observed in higher order calculations in the Weinberg scheme really is to be considered a problem?

We stress again that to really make progress on these issues, it is mandatory to go beyond leading order - nuclear physics can not be described by one-pion exchange plus a certain number of contact terms. In particular, we challenge the authors of Ref. [5] to present a viable and phenomenological successful scheme that could truly be considered an alternative to Weinberg’s original proposal.

Acknowledgments

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